Milan Mroczkowski

VÝRAZY S ODMOCNINAMI S RACIONÁLNÍM EXPONENTEM

YESIT.CZ

KAP. 3/12

$$\sqrt{5} + 3\sqrt{3} - (2\sqrt{5} - 3\sqrt{5} - \sqrt{3}) = \sqrt{5} + 3\sqrt{3} - (-\sqrt{5} - \sqrt{3}) =$$

$$= \sqrt{5} + 3\sqrt{3} + \sqrt{5} + \sqrt{3} =$$

$$= \underline{\underline{2\sqrt{5} + 4\sqrt{3}}}$$

ČÁSTEČNÉ ODMOCŇOVÁNÍ

$$\sqrt{12} = \sqrt{3} \cdot \sqrt{2^2} = \underline{\underline{2\sqrt{3}}}$$

$$\sqrt{50} = \sqrt{2 \cdot 5^2} = \underline{\underline{5\sqrt{2}}}$$

$$\sqrt[3]{48} = \sqrt[3]{6 \cdot 2^3} = \sqrt[3]{6} \cdot \sqrt[3]{2^3} = \underline{\underline{2\sqrt[3]{6}}}$$

$$\sqrt[4]{162} = \sqrt[4]{2 \cdot 81} = \sqrt[4]{2 \cdot 3^4} = \sqrt[4]{2} \cdot \sqrt[4]{3^4} = \underline{\underline{3\sqrt[4]{2}}}$$

$$\sqrt[3]{8a^7 b^8} = \sqrt[3]{2^3 a^{3+4} b^{3+5}} = \sqrt[3]{2^3}\sqrt[3]{a^3 a^4 b^3 b^5} = 2\sqrt[3]{a^3 b^3}\sqrt[3]{a^4 b^5} =$$

$$= 2ab\sqrt[3]{a^{3+1} b^{3+2}} = 2ab\,ab\sqrt[3]{ab^2} =$$

$$= \underline{\underline{2a^2 b^2 \sqrt[3]{ab^2}}}$$

KONTROLA:

$$2a^2 b^2 \sqrt[3]{ab^2} = 2a^{\frac{2}{1}} b^{\frac{2}{1}} a^{\frac{1}{3}} b^{\frac{2}{3}} = 2a^{\frac{2}{1}+\frac{1}{3}} b^{\frac{2}{1}+\frac{2}{3}} = 2a^{\frac{6+1}{3}} b^{\frac{6+2}{3}} = 2a^{\frac{7}{3}} b^{\frac{8}{3}} =$$

$$= 2\sqrt[3]{a^7 b^8} = \sqrt[3]{2^3}\sqrt[3]{a^7 b^8} = \sqrt[3]{2^3 a^7 b^8} = \underline{\sqrt[3]{8a^7 b^8}}$$

$$\sqrt{\sqrt{x}} = \sqrt{x^{\frac{1}{2}}} = \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{1}{4}} = \underline{\underline{\sqrt[4]{x}}}$$

$$\sqrt{\sqrt[3]{x}} = \sqrt{x^{\frac{1}{3}}} = \left(x^{\frac{1}{3}}\right)^{\frac{1}{2}} = x^{\frac{1}{6}} = \underline{\underline{\sqrt[6]{x}}}$$

$$\sqrt[3]{\sqrt[4]{x^3}} = \left(x^{\frac{3}{4}}\right)^{\frac{1}{3}} = x^{\frac{3}{4}\cdot\frac{1}{3}} = x^{\frac{3}{12}} = \sqrt[\cancel{12}\,4]{x^{\cancel{3}}} = \underline{\underline{\sqrt[4]{x}}}$$

$$\sqrt{\frac{25x^2-50x+25}{x^2+4x+4}} = \sqrt{\frac{(5x-5)^2}{(x+2)^2}} = \frac{\sqrt{(5x-5)^2}}{\sqrt{(x+2)^2}} = \underline{\underline{\frac{|5(x-1)|}{|x+2|}}}$$

Pozn.: $(x+2)^2 = 4$ KLADNÉ

dosadím -4

$\sqrt{(x+2)^2} = (-4+2) = -2$ ZÁPORNÉ

KVŮLI ODMOCNINĚ S X JE VÝSLEDEK V ABSOLUTNÍ HODNOTĚ

USMĚRŇOVÁNÍ ZLOMKŮ

$$\frac{3\sqrt{2}}{4\sqrt{5}} = \frac{3\sqrt{2}}{4\sqrt{5}}\cdot 1 = \frac{3\sqrt{2}}{4\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{2\cdot 5}}{4\sqrt{5\cdot 5}} = \frac{3\sqrt{10}}{4\sqrt{25}} =$$

$$= \underline{\underline{\frac{3\sqrt{10}}{20}}}$$

$$\frac{a+\sqrt{b}}{a-\sqrt{b}} - \frac{a-\sqrt{b}}{a+\sqrt{b}} = \frac{a+\sqrt{b}}{a-\sqrt{b}}\cdot 1 - \frac{a-\sqrt{b}}{a+\sqrt{b}}\cdot 1 = \frac{a+\sqrt{b}}{\underset{A-B}{a-\sqrt{b}}}\cdot\frac{a+\sqrt{b}}{\underset{A+B}{a+\sqrt{b}}} - \frac{a-\sqrt{b}}{\underset{A+B}{a+\sqrt{b}}}\cdot\frac{a-\sqrt{b}}{\underset{A-B}{a-\sqrt{b}}} =$$

$$= \frac{(a+\sqrt{b})^2}{\underset{A^2-B^2}{a^2-(\sqrt{b})^2}} - \frac{(a-\sqrt{b})^2}{\underset{A^2-B^2}{a^2-(\sqrt{b})^2}} = \frac{a^2+2a\sqrt{b}+b}{a^2-b} - \frac{a^2-2a\sqrt{b}+b}{a^2-b} =$$

$$= \frac{a^2+2a\sqrt{b}+b-(a^2-2a\sqrt{b}+b)}{a^2-b} = \frac{a^2+2a\sqrt{b}+b-a^2+2a\sqrt{b}-b}{a^2-b} =$$

$$= \underline{\underline{\frac{4a\sqrt{b}}{a^2-b}}}$$

$$\frac{19\sqrt{6}}{9-2\sqrt{6}} = \frac{19\sqrt{6}}{\underset{A-B}{9-2\sqrt{6}}} \cdot \frac{9+2\sqrt{6}}{\underset{A+B}{9+2\sqrt{6}}} = \frac{19\sqrt{6}\cdot 9 + 19\sqrt{6}\cdot 2\sqrt{6}}{\underset{A^2-B^2}{9^2-(2\sqrt{6})^2}} =$$

$$= \frac{19\cdot 9\cdot\sqrt{6} + 19\cdot 2\cdot 6}{81-2^2(\sqrt{6})^2} = \frac{19\cdot 9\cdot\sqrt{6} + 19\cdot 12}{81-24} =$$

$$= \frac{19\cdot 3\cdot 3\cdot\sqrt{6} + 19\cdot 3\cdot 4}{57} = \frac{19\cdot 3\cdot 3\cdot\sqrt{6} + 19\cdot 3\cdot 4}{3\cdot 19} =$$

$$= \frac{\cancel{19\cdot 3}(3\sqrt{6}+4)}{\cancel{19\cdot 3}} = \underline{\underline{3\sqrt{6}+4}}$$

Pokud máme více členů ve jmenovateli, musíme zlomek vynásobit tak, aby ve jmenovateli bylo podle vzorce $(a+b)\cdot(a-b) = a^2-b^2$

IRACIONÁLNÍ ČÍSLA

$$\sqrt{x}\cdot\sqrt[3]{x^2}\cdot\sqrt[4]{x^3} = x^{\frac{1}{2}}\cdot x^{\frac{2}{3}}\cdot x^{\frac{3}{4}} = x^{\frac{1}{2}+\frac{2}{3}+\frac{3}{4}} = x^{\frac{6+8+9}{12}} = x^{\frac{23}{12}} =$$

$$= \underline{\underline{\sqrt[12]{x^{23}}}}$$

$$\left[\left(\frac{a^{\frac{1}{2}}\cdot a^{-\frac{2}{1}}}{a^{\frac{1}{3}}}\right)^{-2}\right]^{\frac{1}{5}} = \left[\left(\frac{a^{-\frac{3}{2}}}{a^{\frac{1}{3}}}\right)^{-2}\right]^{\frac{1}{5}} = \left[\frac{1}{\left(\frac{a^{-\frac{3}{2}}}{a^{\frac{1}{3}}}\right)^2}\right]^{\frac{1}{5}} =$$

$$= \left[\frac{\frac{1}{1}}{\left(\frac{a^{-\frac{3}{2}}}{a^{\frac{1}{3}}}\right)^2}\right]^{\frac{1}{5}} = \left[\frac{1}{1}\cdot\frac{\left(a^{\frac{1}{3}}\right)^2}{\left(a^{-\frac{3}{2}}\right)^2}\right]^{\frac{1}{5}} = \left[\frac{a^{\frac{2}{3}}}{\left(\frac{1}{a^{\frac{3}{2}}}\right)^2}\right]^{\frac{1}{5}} =$$

$$= \left(\frac{a^{\frac{2}{3}}}{1}\cdot\frac{a^{\frac{6}{2}}}{1}\right)^{\frac{1}{5}} = \left(a^{\frac{2}{3}+\frac{6}{2}}\right)^{\frac{1}{5}} = \left(a^{\frac{22}{6}}\right)^{\frac{1}{5}} = a^{\frac{22}{6}\cdot\frac{1}{5}} =$$

$$= a^{\frac{22}{30}} = \underline{\underline{a^{\frac{11}{15}}}}$$

HODNOTA VÝRAZU

a)

$$\frac{1}{x^2} - x^2 \qquad x = 2$$

$$\frac{1}{2^2} - 2^2 = \frac{1}{4} - \frac{4}{1} = \frac{1-16}{4} = \underline{\underline{-\frac{15}{4}}}$$

OPAKOVÁNÍ

$$(a^5b + ab^5) : (a^3b - ab^3) = a^2 + b^2 + \frac{ab^5 + ab^5}{a^3b - ab^3}$$

$$-(a^5b - a^3b^3)$$

$$a^3b^3 + ab^5$$

$$-(a^3b^3 - ab^5)$$

$$ab^5 + ab^5$$

$$a^2 + b^2 + \frac{ab^5 + ab^5}{a^3b - ab^3} =$$

$$= \frac{a^2(a^3b - ab^3) + b^2(a^3b - ab^3) + ab^5 + ab^5}{a^3b - ab^3} =$$

$$= \frac{a^5b - a^3b^3 + a^3b^3 \cancel{- ab^5} \cancel{+ ab^5} + ab^5}{a^3b - ab^3} =$$

$$= \frac{a^5b + ab^5}{a^3b - ab^3} =$$

$$= \frac{\cancel{ab}(a^4 + b^4)}{\cancel{ab}(a^2 - b^2)} =$$

$$= \underline{\underline{\frac{a^4 + b^4}{(a-b)(a+b)}}}$$

$$\frac{a^4-b^4}{a^2+b^2}=\frac{(a^2+b^2)(a^2-b^2)}{a^2+b^2}=\underline{\underline{a^2-b^2}}=(a-b)(a+b)$$

podmínka:

$\underline{a^2+b^2\neq 0}$

podmínky:

$a+b\neq 0 \quad a-b\neq 0$

$\underline{a\neq -b} \quad \underline{a\neq b}$

$$2\sqrt{5}-3\sqrt{5}=\sqrt{5}\,(2-3)=\sqrt{5}\,(-1)=-\sqrt{5}$$

$$3\sqrt{3}+\sqrt{3}=4\sqrt{3}$$

$$\sqrt{5}\cdot\sqrt{5}=5$$

$$2\cdot\sqrt{5}=\sqrt{5}+\sqrt{5}$$

$$\frac{a^2+b^2}{2ab}+1=\frac{a^2+b^2}{2ab}+\frac{2ab}{2ab}=\frac{a^2+b^2+2ab}{2ab}=$$

$$=\frac{a^2+2ab+b^2}{2ab}=$$

$$=\underline{\underline{\frac{(a+b)^2}{2ab}}}$$

$$(1^2-x^2)=(1^2-x^2)=(1-x)(1+x)$$

$$A^2-B^2=\quad A-B\quad A+B$$

$$\left(\frac{3}{-1}\right)^{-1}=\frac{1}{\left(\frac{3}{-1}\right)^1}=\frac{1}{1}\cdot\frac{-1}{3}=\frac{1}{1}\cdot\left(-\frac{1}{3}\right)=\underline{\underline{-\frac{1}{3}}}$$

$$(a^3+b^3):(a+b)=a^2-ab+b^2$$
$$-(a^3+a^2b)$$
$$-a^2b+b^3$$
$$-(-a^2b-ab^2)$$
$$ab^2+b^3$$
$$-(ab^2+b^3)$$
$$0$$

$$(a^3-b^3):(a-b)=a^2+ab+b^2$$
$$-(a^3-a^2b)$$
$$a^2b-b^3$$
$$-(a^2b-ab^2)$$
$$ab^2-b^3$$
$$-(ab^2-b^3)$$
$$0$$

$$\frac{1}{\cancel{(a+b)}}\cdot\frac{(a+b)^2}{2}=\frac{a+b}{2}$$

$$1-a=\sqrt{1-a}\cdot\sqrt{1-a}$$

$$\frac{2ab}{3\cancel{ab}}\cdot\frac{\cancel{ab}}{a^2+b^2}=\frac{2ab}{3(a^2+b^2)}$$

$$x\cdot(-x^2)=-x^3$$

$$\sqrt{2}\cdot\sqrt{5}=\sqrt{2\cdot5}=\sqrt{10}$$

$$\sqrt[3]{x^2}=x^{\frac{2}{3}}$$

$$\frac{a^9}{x}\cdot\frac{x}{a^3}=\frac{a^6\cdot x}{x\cdot1}=a^6$$

ZÁPIS PODMÍNEK

$$x^2 \neq 2 \qquad a \neq \sqrt{b^2}$$

$$x \neq \pm\sqrt{2} \qquad a \neq \pm b$$

$$\frac{\sqrt{(5x-5)^2}}{\sqrt{(x+2)^2}} = \frac{|5(x-1)|}{|x+2|}$$

$$\frac{8b^2c-15a^2c-30abc}{24abc} = \frac{c(8b^2-15a^2-30ab)}{c\,(24ab)} =$$

$$= \frac{8b^2-15a^2-30ab}{24ab}$$

$$\boxed{(a+b)^2} = a^2 + 2ab + b^2$$

nebo také

$$= (a+b)(a+b)$$

$$\boxed{(a-b)^2} = a^2 - 2ab + b^2$$ nebo také $(a-b)(a-b)$

$$(a-b)\cdot(a+b) = \boxed{a^2 - b^2}$$

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$$

ODSTRAŇTE ODMOCNINY ZE JMENOVATELE ZLOMKU (<u>USMĚRNĚTE</u>)

1a $\dfrac{2-\sqrt{3}}{\sqrt{5}}$

1b $\dfrac{5-\sqrt{3}}{5+\sqrt{3}}$

1c $\dfrac{\dfrac{\sqrt{2}-1}{2}-\dfrac{2}{\sqrt{2}+1}}{\sqrt{2}-1+\dfrac{2}{\sqrt{2}+1}}$

1a

$$\frac{2-\sqrt{3}}{\sqrt{5}} = \frac{2-\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}-\sqrt{3}\sqrt{5}}{\sqrt{5\cdot 5}} =$$

$$= \underline{\underline{\frac{\sqrt{5}\,(2-\sqrt{3})}{5}}}$$ nebo také $$\underline{\underline{\frac{2\sqrt{5}-\sqrt{15}}{5}}}$$

1b

$$\frac{5-\sqrt{3}}{5+\sqrt{3}} = \frac{5-\sqrt{3}}{5+\sqrt{3}} \cdot \frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{25-5\sqrt{3}-5\sqrt{3}+3}{25-5\sqrt{3}+5\sqrt{3}-3} =$$

$$= \frac{28-5\sqrt{3}-5\sqrt{3}}{22} = \frac{28-10\sqrt{3}}{22} = \frac{28}{22} - \frac{10\sqrt{3}}{22} =$$

$$= \underline{\underline{\frac{14}{11} - \frac{5\sqrt{3}}{11}}}$$ nebo lépe: $$\underline{\underline{\frac{14-5\sqrt{3}}{11}}}$$

1c

$$\frac{\frac{\sqrt{2}-1}{2} - \frac{2}{\sqrt{2}+1}}{\sqrt{2}-1+\frac{2}{\sqrt{2}+1}} = \frac{\frac{(\sqrt{2}-1)(\sqrt{2}+1)-4}{2(\sqrt{2}+1)}}{\frac{\sqrt{2}-1}{1}+\frac{2}{\sqrt{2}+1}} = \frac{\frac{2+\sqrt{2}-\sqrt{2}-1-4}{2(\sqrt{2}+1)}}{\frac{(\sqrt{2}-1)(\sqrt{2}+1)+2}{\sqrt{2}+1}} =$$

$$= \frac{\frac{-3}{2(\sqrt{2}+1)}}{\frac{2+\sqrt{2}-\sqrt{2}-1+2}{\sqrt{2}+1}} = \frac{-3}{2\cancel{(\sqrt{2}+1)}} \cdot \frac{\cancel{\sqrt{2}+1}}{3} = \frac{-3}{2} \cdot \frac{1}{3} = -\frac{3}{6} =$$

$$= \underline{\underline{-\frac{1}{2}}}$$

UPRAVTE A URČETE PODMÍNKY PRO VŠECHNY PROMĚNNÉ:

2a)

$$\left(\frac{a+1}{\sqrt{a}} + \frac{1}{a-\sqrt{a}} - \frac{a}{\sqrt{a}+1} \right) \cdot \frac{\sqrt{3} - a\sqrt{3}}{a+1} \qquad a > 1$$

2b)

$$\frac{3(xy)^{\frac{1}{n}} - y^{\frac{1}{n}}}{9(xy)^{\frac{2}{n}} - y^{\frac{2}{n}}} \cdot \frac{y^{\frac{1}{n}}}{\left(3x^{\frac{1}{n}}+1\right)^{-2}} \qquad n \in N$$

2a

$$\left(\frac{a+1}{\sqrt{a}}+\frac{1}{a-\sqrt{a}}-\frac{a}{\sqrt{a}+1}\right)\cdot\frac{\sqrt{3}-a\sqrt{3}}{a+1}= \qquad a>1$$

$$=\left(\frac{a+1}{\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}}+\frac{1}{a-\sqrt{a}}\cdot\frac{a+\sqrt{a}}{a+\sqrt{a}}-\frac{a}{\sqrt{a}+1}\cdot\frac{\sqrt{a}-1}{\sqrt{a}-1}\right)\cdot\frac{\sqrt{3}\,(1-a)}{a+1}=$$

$$=\left(\frac{a\sqrt{a}+\sqrt{a}}{(\sqrt{a})^2}+\frac{a+\sqrt{a}}{(a)^2-(\sqrt{a})^2}-\frac{a\sqrt{a}-a}{(\sqrt{a})^2-(1)^2}\right)\cdot\frac{\sqrt{3}\,(1-a)}{a+1}=$$

$$=\left(\frac{a\sqrt{a}+\sqrt{a}}{a}+\frac{a+\sqrt{a}}{a^2-a}-\frac{a\sqrt{a}-a}{a-1}\right)\cdot\frac{\sqrt{3}\,(1-a)}{a+1}=$$

$$=\left(\frac{a\sqrt{a}+\sqrt{a}}{a}+\frac{a+\sqrt{a}}{a(a-1)}-\frac{a\sqrt{a}-a}{a-1}\right)\cdot\frac{\sqrt{3}\,(1-a)}{a+1}=$$

$$=\left(\frac{(a\sqrt{a}+\sqrt{a})(a-1)+a+\sqrt{a}-a(a\sqrt{a}-a)}{a(a-1)}\right)\cdot\frac{\sqrt{3}\,(-1)(a-1)}{a+1}=$$

$$=\left(\frac{\cancel{a^2\sqrt{a}}\cancel{-a\sqrt{a}}\cancel{+a\sqrt{a}}-\sqrt{a}+a+\sqrt{a}\cancel{-a^2\sqrt{a}}+a^2}{a(a-1)}\right)\cdot\frac{\sqrt{3}\,(-1)(a-1)}{a+1}=$$

$$=\frac{a+a^2}{a\,\cancel{(a-1)}}\cdot\frac{-\sqrt{3}\,\cancel{(a-1)}}{a+1}=$$

$$=\frac{a^2+a}{a}\cdot\frac{-\sqrt{3}}{a+1}=$$

$$=\frac{a\,\cancel{(a+1)}}{a}\cdot\frac{-\sqrt{3}}{\cancel{a+1}}=\frac{a}{a}\cdot(-\sqrt{3})=1\cdot(-\sqrt{3})=\underline{\underline{-\sqrt{3}}}$$

Podmínky: $\underline{a\neq 0}$ $\quad a+1\neq 0$, $\underline{a\neq -1}$ $\quad a-1\neq 0$, $\underline{a\neq 1}$

(2b)

$$\frac{3(xy)^{\frac{1}{n}} - y^{\frac{1}{n}}}{9(xy)^{\frac{2}{n}} - y^{\frac{2}{n}}} \cdot \frac{y^{\frac{1}{n}}}{\left(3x^{\frac{1}{n}}+1\right)^{-2}} = \qquad n \in N$$

$$= \frac{3x^{\frac{1}{n}}y^{\frac{1}{n}} - y^{\frac{1}{n}}}{9x^{\frac{2}{n}}y^{\frac{2}{n}} - y^{\frac{2}{n}}} \cdot \frac{y^{\frac{1}{n}}}{\frac{1}{\left(3x^{\frac{1}{n}}+1\right)^{2}}} =$$

$$= \frac{3x^{\frac{1}{n}}y^{\frac{1}{n}} - y^{\frac{1}{n}}}{3^2x^{\frac{2}{n}}y^{\frac{2}{n}} - y^{\frac{2}{n}}} \cdot y^{\frac{1}{n}} \cdot \left(3x^{\frac{1}{n}}+1\right)^{2} =$$

$$= \frac{y^{\frac{1}{n}}\left(3x^{\frac{1}{n}}-1\right)}{\underbrace{\left(3x^{\frac{1}{n}}y^{\frac{1}{n}}\right)^2 - \left(y^{\frac{1}{n}}\right)^2}_{A^2 - B^2}} \cdot y^{\frac{1}{n}} \cdot \left(3x^{\frac{1}{n}}+1\right)\left(3x^{\frac{1}{n}}+1\right) =$$

$$= \frac{y^{\frac{1}{n}}\left(3x^{\frac{1}{n}}-1\right)}{\left(3x^{\frac{1}{n}}y^{\frac{1}{n}} - y^{\frac{1}{n}}\right)\left(3x^{\frac{1}{n}}y^{\frac{1}{n}} + y^{\frac{1}{n}}\right)} \cdot y^{\frac{1}{n}} \cdot \left(3x^{\frac{1}{n}}+1\right)\left(3x^{\frac{1}{n}}+1\right) =$$

$$= \frac{\cancel{y^{\frac{1}{n}}\left(3x^{\frac{1}{n}}-1\right)}}{\cancel{y^{\frac{1}{n}}\left(3x^{\frac{1}{n}}-1\right)}\,\cancel{y^{\frac{1}{n}}}\cancel{\left(3x^{\frac{1}{n}}+1\right)}} \cdot \frac{\cancel{y^{\frac{1}{n}}} \cdot \cancel{\left(3x^{\frac{1}{n}}+1\right)}\left(3x^{\frac{1}{n}}+1\right)}{1} =$$

$$= \underline{\underline{3x^{\frac{1}{n}}+1}}$$

$\underline{x \neq 0}$

$\underline{y \neq 0}$

PRO KLADNÉ ZÁKLADY MOCNIN
ZJEDNODUŠTE :

3a

$$\left(\frac{n^{\frac{1}{3}}}{n^{\frac{1}{2}} \cdot n^{-1}}\right)^{\frac{3}{4}}$$

3b

$$\frac{\left(a^{\frac{3}{4}} b^{-\frac{2}{3}}\right)^{-\frac{1}{2}}}{\left(a^{\frac{1}{2}} b^{-\frac{2}{3}}\right)^{-\frac{3}{4}}}$$

3a

$$\left(\frac{m^{\frac{1}{3}}}{m^{\frac{1}{2}}\cdot m^{-1}}\right)^{\frac{3}{4}} = \frac{m^{\frac{3}{12}}}{m^{\frac{3}{8}}\cdot m^{-\frac{3}{4}}} =$$

$$= \frac{m^{\frac{3}{12}}}{m^{\frac{3}{8}}\cdot \frac{1}{m^{\frac{3}{4}}}} = \frac{\frac{m^{\frac{3}{12}}}{1}}{\frac{m^{\frac{3}{8}}}{m^{\frac{3}{4}}}} = \frac{m^{\frac{3}{12}}}{1}\cdot\frac{m^{\frac{3}{4}}}{m^{\frac{3}{8}}} =$$

$$= \frac{m^{\frac{12}{12}}}{m^{\frac{3}{8}}} = \frac{m}{m^{\frac{3}{8}}} = m^{\frac{1}{1}} : m^{\frac{3}{8}} = m^{\frac{8-3}{8}} = \underline{\underline{m^{\frac{5}{8}}}}$$

$m \neq 0$

TO SAMÉ, JINÝ POSTUP:

$$\left(\frac{m^{\frac{1}{3}}}{m^{\frac{1}{2}}\cdot m^{-1}}\right)^{\frac{3}{4}} = \left(\frac{m^{\frac{1}{3}}}{m^{\frac{1}{2}-1}}\right)^{\frac{3}{4}} = \left(\frac{m^{\frac{1}{3}}}{m^{-\frac{1}{2}}}\right)^{\frac{3}{4}} = \left(\frac{m^{\frac{1}{3}}}{\frac{1}{m^{\frac{1}{2}}}}\right)^{\frac{3}{4}} = \left(m^{\frac{1}{3}}\cdot m^{\frac{1}{2}}\right)^{\frac{3}{4}} =$$

$$= \left(m^{\frac{2+3}{6}}\right)^{\frac{3}{4}} = \left(m^{\frac{5}{6}}\right)^{\frac{3}{4}} = m^{\frac{5}{6}\cdot\frac{3}{4}} = m^{\frac{15}{24}} = \underline{\underline{m^{\frac{5}{8}}}}$$

$m \neq 0$

Poznámka mimo příklad:

$$\frac{d^m}{d^n} = d^{m-n} \qquad \frac{m^{\frac{1}{2}}}{m^{\frac{2}{1}}} = m^{\frac{1}{2}-\frac{2}{1}} = m^{\frac{1-4}{2}} = m^{-\frac{3}{2}} = \frac{1}{m^{\frac{3}{2}}}$$

3b

$$\frac{\left(a^{\frac{3}{4}} b^{-\frac{2}{3}}\right)^{-\frac{1}{2}}}{\left(a^{\frac{1}{2}} b^{-\frac{2}{3}}\right)^{-\frac{3}{4}}} = \frac{\left(\dfrac{a^{\frac{3}{4}}}{b^{\frac{2}{3}}}\right)^{-\frac{1}{2}}}{\left(\dfrac{a^{\frac{1}{2}}}{b^{\frac{2}{3}}}\right)^{-\frac{3}{4}}} =$$

$$= \frac{\dfrac{1}{\left(\dfrac{a^{\frac{3}{4}}}{b^{\frac{2}{3}}}\right)^{\frac{1}{2}}}}{\dfrac{1}{\left(\dfrac{a^{\frac{1}{2}}}{b^{\frac{2}{3}}}\right)^{\frac{3}{4}}}} = \frac{\dfrac{1}{\dfrac{a^{\frac{3}{8}}}{b^{\frac{2}{6}}}}}{\dfrac{1}{\dfrac{a^{\frac{3}{8}}}{b^{\frac{6}{12}}}}} = \frac{\dfrac{b^{\frac{2}{6}}}{a^{\frac{3}{8}}}}{\dfrac{b^{\frac{6}{12}}}{a^{\frac{3}{8}}}} = \frac{b^{\frac{2}{6}}}{a^{\frac{3}{8}}} \cdot \frac{a^{\frac{3}{8}}}{b^{\frac{6}{12}}} =$$

$$= \frac{b^{\frac{2}{6}}}{b^{\frac{6}{12}}} = \frac{b^{\frac{1}{3}}}{b^{\frac{1}{2}}} = b^{\frac{1}{3}} : b^{\frac{1}{2}} = b^{\frac{2-3}{6}} = b^{-\frac{1}{6}} = \frac{1}{b^{\frac{1}{6}}} = \underline{\underline{\frac{1}{\sqrt[6]{b}}}}$$

$b \neq 0$

TO SAMÉ, ALE JINÝ POSTUP

$$\frac{\left(a^{\frac{3}{4}} b^{-\frac{2}{3}}\right)^{-\frac{1}{2}}}{\left(a^{\frac{1}{2}} b^{-\frac{2}{3}}\right)^{-\frac{3}{4}}} = \frac{\left[a^{\frac{3}{4}\cdot\left(-\frac{1}{2}\right)} b^{-\frac{2}{3}\cdot\left(-\frac{1}{2}\right)}\right]}{a^{\frac{1}{2}\cdot\left(-\frac{3}{4}\right)} b^{\left(-\frac{2}{3}\right)\cdot\left(-\frac{3}{4}\right)}} = \frac{\cancel{a^{-\frac{3}{8}}}\, b^{\frac{1}{3}}}{\cancel{a^{-\frac{3}{8}}}\, b^{\frac{1}{2}}} = \frac{b^{\frac{1}{3}}}{\boxed{b^{\frac{1}{2}}}} =$$

$$= \frac{b^{\frac{1}{3}}}{\dfrac{1}{b^{-\frac{1}{2}}}} = b^{\frac{1}{3}-\frac{1}{2}} = b^{\frac{2-3}{6}} = \underline{\underline{b^{-\frac{1}{6}}}}$$ nebo $\frac{1}{b^{\frac{1}{6}}}$

nebo $\frac{1}{\sqrt[6]{b}}$

$$b^{\frac{1}{2}} = \frac{1}{b^{-\frac{1}{2}}} = \sqrt{b}$$

$$b^{-\frac{1}{2}} = \frac{1}{b^{\frac{1}{2}}} = \frac{1}{\sqrt{b}}$$

PRO KLADNÉ ZÁKLADY ODMOCNIN ZJEDNODUŠTE A VYJÁDŘETE JAKO MOCNINY:

4a) $\sqrt{\dfrac{a\sqrt[3]{b}}{\sqrt[3]{a\sqrt{b}}}}$

4a

$$\sqrt{\frac{a\sqrt[3]{b}}{\sqrt[3]{a\sqrt{b}}}} = \left(\frac{a\cdot b^{\frac{1}{3}}}{\left(a\cdot b^{\frac{1}{2}}\right)^{\frac{1}{3}}}\right)^{\frac{1}{2}} = \left(\frac{a\cdot b^{\frac{1}{3}}}{a^{\frac{1}{3}}\cdot b^{\frac{1}{6}}}\right)^{\frac{1}{2}} = \frac{\left(a\cdot b^{\frac{1}{3}}\right)^{\frac{1}{2}}}{\left(a^{\frac{1}{3}}\cdot b^{\frac{1}{6}}\right)^{\frac{1}{2}}} =$$

$$= \frac{a^{\frac{1}{2}}\ b^{\frac{1}{3}\cdot\frac{1}{2}}}{a^{\frac{1}{3}\cdot\frac{1}{2}}\ b^{\frac{1}{6}\cdot\frac{1}{2}}} = \frac{a^{\frac{1}{2}}\ b^{\frac{1}{6}}}{a^{\frac{1}{6}}\ b^{\frac{1}{12}}} = \frac{a^{\frac{1}{2}}}{a^{\frac{1}{6}}}\cdot\frac{b^{\frac{1}{6}}}{b^{\frac{1}{12}}} =$$

$$= \frac{a^{\frac{1}{2}}}{\frac{1}{a^{-\frac{1}{6}}}}\cdot\frac{b^{\frac{1}{6}}}{\frac{1}{b^{-\frac{1}{12}}}} = a^{\frac{1}{2}}\cdot a^{-\frac{1}{6}}\cdot b^{\frac{1}{6}}\cdot b^{-\frac{1}{12}} =$$

$$= a^{\frac{3-1}{6}}\cdot b^{\frac{2-1}{12}} = a^{\frac{2}{6}}\cdot b^{\frac{1}{12}} = \underline{\underline{a^{\frac{1}{3}}\cdot b^{\frac{1}{12}}}}$$